5/19/17 "Success is a journey, not a destination."-Ben Sweetland

HW: "Conditional Probability" homework section (due Tuesday 5/23)
June 2016 #25-37 (skip #26) due Tuesday 5/23
Test 2 on Monday

AIM: What is Conditional Probability?

Warm Up:

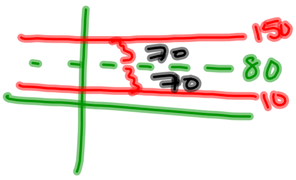
1) The Ferris wheel at the landmark Navy Pier in Chicago takes 7 minutes to make one full rotation. The height, H, in feet, above the ground of one of the six-person cars can be modeled by $H(t)=70\sin\left(\frac{2\pi}{7}(t-1.75)\right)+80$, where t is time, in minutes. Find the car's minimum and maximum heights, in feet..

Period

A

C

$$\text{Min} = C-|A| = 80-70 = 10 \text{ ft}$$



$$\text{Max} = C+|A| = 80+70 = 150 \text{ ft.}$$

When the probability of one event occurring changes depending on other events occurring then we say that there is a **conditional probability**. The language and symbolism of conditional probability can be a bit confusing, but the idea is fairly straightforward and can be developed with two-way frequency charts.

***Exercise* #1:** Let's revisit a two-way frequency chart we saw in the last lesson. In this study, 52 graduating seniors were surveyed as to their post-graduation plans and then the results were sorted by gender.

Let the following letters stand for the following events.

M = Male F = Female

C = Going to College N = Not going to college

	Gender		
	Male	Female	Total
Going to College	16	13	29
Not Going to College	14	9	23
Total	30	22	52

If a person was picked at random, find the probability that the person was

(a) a female, i.e. $P(\text{F})$

$$\frac{\text{want}}{\text{total}} = \frac{22}{52} \text{ or } .42$$

42%

(b) going to college $P(\text{C})$ $= \frac{29}{52}$ or .56

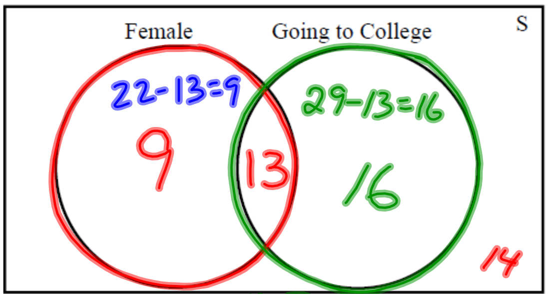
56%

(c) going to college **given** they are female, i.e. $P(\text{C} \mid \text{F})$. Draw a Venn diagram below to help justify the ratio that you give as the probability.

↑ Given

$$P(C|F) = \frac{13}{22} = .59$$

59%



$9+13+16=38$

$52-38=14$

↑ Left over in "universe"

(d) Which is more likely, that a person picked at random will be going to college, given they are a male, i.e. $P(\text{C} \mid \text{M})$, or that a person will be male, given they are going to college, i.e. $P(\text{M} \mid \text{C})$. Show that calculations for both.

2nd (what to look for) / 1st where you're looking

$P(\text{C} \mid \text{M})$

$$= \frac{16}{30 \text{ males}} \approx .53$$

$$\frac{\text{College male}}{\text{males}}$$

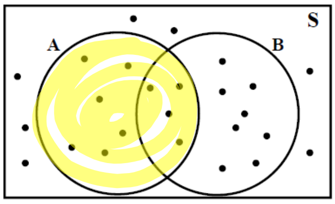
$P(\text{M} \mid \text{C})$

$$= \frac{16}{29 \text{ going to college}} \approx .55$$

$$\frac{\text{Males in college}}{\text{All College}}$$

We can generalize this process to calculate these conditional probabilities based on counts and a way to calculate these probabilities based on other probabilities.

***Exercise* #2:** In the generic Venn diagram shown to the right. Each dot represents an equally likely outcome of the sample space. Some of these fall only into event A, some only into event B, some in both events and some in neither.



(a) Consider the probability of A occurring given that B has occurred. Give a formula for this probability based on counting the number of elements in each set and their intersection.

$P(A \mid B) = \frac{4}{12} = \frac{1}{3} \approx .33$ or 33%

$$\boxed{P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}}$$

Formula for Conditional Probability

***Exercise* #3:** A survey was taken to examine the relationship between hair color and eye color. The chart below shows the proportion of the people surveyed who fell into each category. If a person was picked at random, find each of the following conditional probabilities. Show the calculation you used.

		Hair Color			
		Black	Blond	Red	Total
Eye Color	Blue	0.15	0.20	0.05	0.40
	Brown	0.25	0.10	0.00	0.35
	Green	0.05	0.05	0.15	0.25
	Total	0.45	0.35	0.20	1.00

(a) Find the probability the person picked had brown eyes given they had blond hair.

$P(\text{brown eyes} \mid \text{blond hair})$

$\frac{P(\text{Both})}{P(\text{blond})} = \frac{.10}{.35} = \frac{2}{7}$ or 29%

(b) Find the probability the person had red hair given they had green eyes.

$P(\text{red hair} \mid \text{green eyes})$

$\frac{P(\text{Both})}{P(\text{Green})} = \frac{.15}{.25} = \frac{3}{5}$ 60%

(c) Does having red hair seem have some **dependence** on having green eyes? How can you tell or quantify this dependence?

$P(\text{red}) = .20$ 20%

$P(\text{red} \mid \text{green}) = .60$ 60%

3 times more likely to have red hair if you have green eyes.